Bernoulli Family: Discrete, independent trials, each with probability of success p (and probability of failure $q = 1-p$). Example: Repeatedly flipping a coin, or sampling from an infinite population.

Number of successes in n attempts:

$$\mathcal{Bin}(n,p) \ - \ P(X=k) = \binom{n}{k} p^k q^{n-k}$$
where $k \in \{0,1,2,\ldots,n\}$
Mean $\mu = np$, variance $\sigma^2 = npq$.

If n is large and p is small, then $\mathcal{Bin}(n,p) \doteq \mathcal{P}(np)$.

Number of attempts before rth success:

$$\mathcal{NegBin}(r,p) \ - \ P(X=k) = \binom{k-1}{r-1} p^r q^{k-r}$$
where $k \in \{r, r+1, r+2\ldots\}$
Mean $\mu = r/p$, variance $\sigma^2 = rq/p^2$.

Number of attempts before 1st success:

$$\mathcal{G}(p) = \mathcal{NegBin}(1,p) \ - \ P(X=k) = pq^{k-1}$$

Hypergeometric Family: Discrete, dependent trials, in which each success decreases the future probability of success. Example: Sampling from a finite population (say, choosing n items from N, of which M are considered "successes").

Number of successes in n attempts:

$$\mathcal{H}(N,n,M) \ - \ P(X=k) = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$$
where $k \in \{0,1,2,\ldots,n\}$
$$\mu = n\left(\tfrac{M}{N}\right), \quad \sigma^2 = n\left(\tfrac{M}{N}\right)\left(\tfrac{N-M}{N}\right)\left(\tfrac{N-n}{N-1}\right).$$

If $N >> n$, then $\mathcal{H}(N,n,M) \doteq \mathcal{Bin}(n, M/N)$.

Number of attempts before rth success:

$$P(X=k) = \frac{\binom{k-1}{r-1}\binom{N-k}{M-r}}{\binom{N}{M}}$$
where $k \in \{r, r+1, r+2\ldots, N-M+r\}$
$$\mu = r\left(\tfrac{N+1}{M+1}\right), \quad \sigma^2 = r\left(\tfrac{N+1}{M+1}\right)\left(\tfrac{N-M}{M+1}\right)\left(\tfrac{M+1-r}{M+2}\right).$$

(This is an example of a *Pòlya* distribution, related to Pòlya's urn.)
When $N, M >> r$, this may be approximated by a $\mathcal{NegBin}(r, M/N)$ distribution.

Poisson Family: Events occur at irregular intervals. Events in non-overlapping intervals are independent. The probability of *one* event occurring in a small interval is proportional to the length of the interval; the probability of *more than one* event in a small interval is negligible.

Number of events ("successes") in an interval of length t:

$$\mathcal{P}(\lambda t) \ - \ P(X=k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$
where $k \in \{0,1,2,\ldots\}$
$$\mu = \sigma^2 = \lambda t.$$

Length of the interval until the rth event ("success"):

$$\mathcal{Gamma}(r, 1/\lambda) \ - \ f(t) = \frac{\lambda(\lambda t)^{r-1} e^{-\lambda t}}{(r-1)!}$$
where $t > 0$
$$\mu = r/\lambda, \quad \sigma^2 = r/\lambda^2.$$

Length of the interval until the 1st event:

$$\mathcal{E}(\lambda) = \mathcal{Gamma}(1, 1/\lambda) \ - \ f(t) = \lambda e^{-\lambda t}$$